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By

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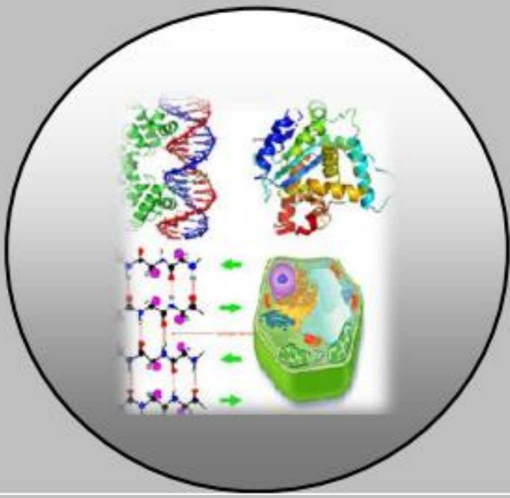
ISSN 2319-3077 Online/Electronic
ISSN 0970-4973 Print

J. Biol. Chem. Research
Volume 30 (2) 2013 Pages No. 1006-1012



Journal of Biological and Chemical Research

An International Peer Reviewed / Referred Journal of Life Sciences and Chemistry

Indexed and Abstracted: Index Copernicus International (Europe), Chemical Abstract Services (U.S.A.), Indian Science.in, Open J-Gate, Database Electronic Journal Library, Google Scholar, e-Journal Portal, InfoBase Index and Eye Source

Published by Society for Advancement of Sciences®

J. Biol. Chem. Research. Vol. 30, No. 2: 1006-1012 (2013)
(An International Peer reviewed Journal of Life Sciences and Chemistry)
Ms 30/2/140/2013

ISSN 0970-4973 (Print)
ISSN 2319-3077 (Online/Electronic)

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RESEARCH PAPER

Received: 18/10/2013 Revised: 30/11/2013 Accepted: 01/12/2013

Estimation of Population Mean Inverse inside the Index of Reliability in Health Sciences

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ABSTRACT

The coefficient of variation (CV) is moderately used as an index of reliability or variability in the health sciences. But the estimation of population coefficient of variance (CV) is a difficult task because of involvement of inverse of mean $\left(\frac{1}{\mu}\right)$. *This paper presents two generalized classes of estimators for the estimation of inverse of population mean* $\left(\frac{1}{\mu}\right)$ *in two different situations (1) unknown and (ii) known population variance* σ^2 *. Expressions for bias and mean squared error of the proposed estimators are obtained and their properties are analyzed in the context of large sample approximations. Further, sub-classes of optimum estimators in the sense of having minimum mean squared error are found. All the results of previous particular estimators may easily be derived as special case of this study.*
Keywords: *Population, Inverse, Reliability and Health Sciences.*

INTRODUCTION

The coefficient of variation (CV) is moderately used as an index of reliability or variability in the health sciences (medical and biological sciences) for the purpose of clinical research and clinical practice in the context of diagnostic tests, human performance tests, and biochemical laboratory assays [Methods of Clinical Epidemiology, 2013].

But the estimation of population coefficient of variance (CV) is a difficult task because of involvement of inverse of mean. Let we discuss some technique to estimate inverse of mean.

ESTIMATION OF INVERSE OF MEAN

Quite often in many situations of practical importance the problem of estimation of the inverse of population mean arises. For instance, if we estimate the coefficient of variation by the ratio of sample standard deviation to sample mean, in such case the estimator may not possess finite moments owing to presence of sample mean in the denominator. Similarly, it is well known that the inverse of sample mean $\left(\frac{1}{\bar{y}}\right)$ is a maximum likelihood estimator of the inverse of population mean $\left(\frac{1}{\mu}\right)$ in the context of normal population but it does not have finite moments. There are also many other instances where the inverse of population mean is the parameter of interest, e.g., in econometrics and biological sciences ; see zellner (1978).

Suppose the square of coefficient of variation $C^2 = \left(\frac{\sigma^2}{\mu^2}\right)$ is known then the family of estimators for estimating the inverse of population mean $\left(\frac{1}{\mu}\right)$ with characterizing scalars *g* and *k* [3] is given by

$$d = \frac{1}{\bar{y}}\left[\frac{1+g\frac{C^2}{n}}{1+k\frac{C^2}{n}}\right] = \frac{1}{\bar{y}}\left[\left(1+g\frac{C^2}{n}\right)\left(1+k\frac{C^2}{n}\right)^{-1}\right] \quad ……(1.1)$$

when C^2 is not known then it is to be estimated by any of the two sample consistent estimators in the following two different situations :

a) $\hat{C}^2 = \frac{s^2}{\bar{y}^2}, \hat{C}^2 = \frac{s^2}{\bar{y}^2}\left(1-\frac{s^2}{n\bar{y}^2}\right)^{-1}$ when σ^2 is unknown

and

b) $\hat{C}^2 = \frac{\sigma^2}{\bar{y}^2}, \hat{C}^2 = \frac{\sigma^2}{\bar{y}^2}\left(1-\frac{\sigma^2}{n_{\bar{y}}^2}\right)^{-1}$ when σ^2 is known,

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y-\bar{y})^2$ are unbiased estimators of population mean μ and variance σ^2 respectively based on a random sample $y_1, y_2, ..., y_n$ of size *n*.

Substituting C^2 in (1.1) leads to the various estimators considered by the previous authors. Keeping in view, the form of the previous estimators, we propose the following generalized family of estimators :

(i) when σ^2 unknown

$$t = \frac{1}{\bar{y}} f\left(\frac{s^2}{n\bar{y}^2}\right) = \frac{1}{\bar{y}} f(u),\ u = \frac{s^2}{n\bar{y}^2}$$

(ii) when σ^2 known

$$t^* = \frac{1}{\bar{y}} g\left(\frac{\sigma^2}{n\bar{y}^2}\right) = \frac{1}{\bar{y}} g(v),\ v = \frac{\sigma^2}{n\bar{y}^2}$$

where *f* (*u*) and *g*(*v*) are functions of *u* and *v* respectively, such that *f*(0) = 1 anf *g*(0) = 1 satisfying the validity conditions of Maclaurin's (Taylor's) series expansion.

It may be easily seen that when is unknown the estimators considered by :

1. Srivastava and Bhatnagar (1981): with *k* being the characterizing scalar

$$d_1 = \frac{\bar{y}}{\bar{y}^2 + k\frac{s^2}{n}} = \frac{1}{\bar{y}}\left(1 + k\frac{s^2}{n\bar{y}^2}\right)^{-1}$$

For

$$f(u) = (1 + ku)^{-1}$$

2. Bhatnagar (1981) :[3] with *g* and *k* being the characterizing scalars

$$d_2 = \frac{1}{\bar{y}}\left[\frac{\bar{y}^2 + g\frac{s^2}{n}}{\bar{y}^2 + k\frac{s^2}{n}}\right] = \frac{1}{\bar{y}}\left[\left(1 + g\frac{s^2}{n\bar{y}^2}\right)\left(1 + k\frac{s^2}{n\bar{y}^2}\right)^{-1}\right]$$

For

$$f(u) = \left[(1 + gu)\,(1 + ku)^{-1}\right]$$

are the special cases of the proposed generalized family of estimators *t*.

The relative bias (RB) and relative mean squared error (RMSE) of the above estimators d_1 and d_2 upto the terms of order $O(n^{-2})$ for symmetric population are given as follow :

THE RELATIVE BIASES AND RELATIVE MEAN SQUARED ERRORS OF d_1 :

$$RB(d_1) = \frac{c^2}{n}\left[(1 - k) + \{k(k - 6) + 3\}\frac{C^2}{n}\right]$$

$$RMSE\,(d_1) = \frac{C^2}{n}\left[1 + \{k(k - 8) + 9\}\frac{C^2}{n}\right]$$

THE RELATIVE BIASES AND RELATIVE MEAN SQUARED ERRORS OF d_2 :

$$RB(d_2) = \frac{c^2}{n}\left[(g - k + 1) + \{(g - k)(6 - k) + 3\}\frac{C^2}{n}\right]$$

$$RMSE\,(d_2) = \frac{C^2}{n}\left[1 + \{k(g - k)(g - k + 8) + 9\}\frac{C^2}{n}\right]$$

Further, when σ^2 is known, the following estimators considered by :

1. Srivastava and Bhatnagar (1981) : With *g* being the characterizing scalar

$$d_1^* = \frac{\bar{y}}{\bar{y}^2 + g\frac{\sigma^2}{n}} = \frac{1}{\bar{y}}\left(1 + g\frac{\sigma^2}{n\bar{y}^2}\right)^{-1}$$

For

$$g(v) = (1 + gv)^{-1}$$

2. Bhatnagar (1981) :[3] With *g* and *k* being the characterizing scalars

$$d_2^* = \frac{1}{\bar{y}}\left[\frac{\bar{y}^2 + g\frac{\sigma^2}{n}}{\bar{y}^2 + k\frac{\sigma^2}{n}}\right] = \frac{1}{\bar{y}}\left[\left(1 + g\frac{\sigma^2}{n\bar{y}^2}\right)\left(1 + k\frac{\sigma^2}{n\bar{y}^2}\right)^{-1}\right]$$

For

$$g(v) = \left[(1 + gv)\,(1 + kv)^{-1}\right]$$

are the special cases of the proposed generalized family of estimators *t**

The relative bias (RB) and relative mean squared error (RMSE) of the above estimators d_1^* and d_2^* upto the terms of order $O(n^{-2})$ for symmetric population are given as follow :

THE RELATIVE BIASES AND RELATIVE MEAN SQUARED ERRORS OF d_1^* :

$$RB\left(d_1^*\right) = RB\frac{C^2}{n}\left[(1-g) + \{g(g-6)+3\}\frac{C^2}{n}\right]$$

$$RMSE\left(d_1^*\right) = \frac{C^2}{n}\left[1 + \{g(g-8)+9\}\frac{C^2}{n}\right]$$

THE RELATIVE BIASES AND RELATIVE MEAN SQUARED ERRORS OF d_2^* :

$$RB\left(d_2^*\right) = \frac{C^2}{n}\left[(g-k+1) + \{(g-k)(6-k)+3\}\frac{C^2}{n}\right]$$

$$RMSE\left(d_2^*\right) = \frac{C^2}{n}\left[1 + \{(g-k)(g-k+8)+9\}\frac{C^2}{n}\right]$$

THE RELATIVE BIASES AND RELATIVE MEAN SQUARED ERRORS OF *t* AND *t**

In order to evaluate the relative bias and relative mean squared error of *t* and *t** upto the terms of order $O(n^{-2})$ under large sample approximation, we introduce the following notations

$$u = (\bar{y} - \mu)/\mu, \quad v = \left(s^2 - \sigma^2\right)/\mu^2 \qquad \text{……(2.1)}$$

Where *u* and *v* are order $O(n^{-1/2})$ in probability with

E (U) = E(V) = 0. Expanding $t=\frac{1}{\bar{y}}f(u)$ in a third order Taylor's (Maclaurin's) series about the point u = 0, with f '(0), f '' (0) and f'''(0) being the first, second and third derivatives respectively, we have

$$t=\frac{1}{\bar{y}}\left[f(0)+uf'(0)+\frac{\mu^2}{2!}f''(0)+\frac{\mu^3}{3!}f'''(\mu^*)\right]$$

Or,

$$t=\frac{1}{\bar{y}}\left[1+\frac{s^2}{n\bar{y}^2}f'(0)+\frac{1}{2n^2}\left(\frac{s^2}{\bar{y}^2}\right)^2 f''(0)+\frac{u^3}{3!}f'''(\mu^*)\right]$$

……(2.2)

Where $u^*=hu$, 0 < h < 1.

From (2.1) and (2.2) we have

$$\frac{\left(t-\frac{1}{\mu}\right)}{\frac{1}{\mu}}=\left[-U(1+U)^{-1}+\frac{1}{n}\left\{\left(V+C^2\right)(1+U)^{-3}f'(0)+\frac{1}{2n}\left(V+C^2\right)^2(1+U)^{-5}f''(0)\right\}\right]$$

……(2.3)

Expanding the right hand side of (2.3) and retaining terms of order $O(n^{-2})$ in probability, we find

$$\frac{\left(t-\frac{1}{\mu}\right)}{\frac{1}{\mu}}=e_{-1/2}+e_{-1}+e_{-3/2}+e_{-2}$$

……(2.4)

Where,

$$\left.\begin{aligned}
e_{-1/2}&=-U\\
e_{-1}&=U^2+\frac{C^2}{n}f'(0)\\
e_{-3/2}&=-U^3+\frac{1}{n}\left(v-3c^2U\right)f'(0)\\
e_{-2}&=U^4+\frac{1}{n}\left[\left(6c^2U^2-3UV\right)f'(0)+\frac{C^4}{2n}f''(0)\right]
\end{aligned}\right\}$$

……(2.5)

Here the suffixes of e indicate the order of n.

From (2.4), it is easy to verify that the relative bias and relative mean squared error upto the terms of order $O(n^{-2})$ are

$$RB(t)=\frac{C^2}{n}\left[1+f'(0)+\left\{6f'(0)+\frac{f''(0)}{2}+3-\theta(1+3f'(0))\right\}\frac{C^2}{n}\right]$$

……(2.6)

and

$$RMSE(t)=\frac{C^2}{n}\left[1+\left\{f'(0)\left(f'(0)+8\right)+9-2\theta\left(f'(0)+1\right)\right\}\frac{C^2}{n}\right]$$

……(2.7)

Where $\theta = \gamma_1 / C$, γ_1 is the Pearson's measure of skewness of the population.

Proceeding on similar lines as above, with *g*(0), *g*''(0) and *g*'''(0) being the first, second and third derivatives ar the point *v* = 0 the relatives bias and relative mean squared error of *t** upto the terms of order $O(n^{-2})$ are

$$RB(t^*) = \frac{C^2}{n}\left[1 + g'(0) + \left\{6g'(0) + \frac{g''(0)}{2} + 3 - \theta\right\}\frac{C^2}{n}\right] \qquad \text{……(2.8)}$$

and

$$RMSE(t^*) = \frac{C^2}{n}\left[1 + \{g'(0)(g'(0)+8) + 9 - 2\theta\}\frac{C^2}{n}\right] \qquad \text{……(2.9)}$$

If the population is symmetrical, i.e., $\theta = 0$ the expression from (2.6) to (2.9) further reduce to

$$RB(t) = \frac{C^2}{n}\left[1 + f'(0) + \left\{6f'(0) + \frac{f''(0)}{2} + 3\right\}\frac{C^2}{n}\right] \qquad \text{……(2.10)}$$

$$RMSE(t) = \frac{C^2}{n}\left[1 + \left\{f'(0)\left(f'(0)+8\right) + 9\right\}\frac{C^2}{n}\right] \qquad \text{……(2.11)}$$

and

$$RB(t^*) = \frac{C^2}{n}\left[1 + g'(0)\left\{6g'(0) + \frac{g''(0)}{2} + 3\right\}\frac{C^2}{n}\right] \qquad \text{……(2.12)}$$

$$RMSE(t^*) = \frac{C^2}{n}\left[1 + \left\{g'(0)\left(g'(0)+8\right) + 9\right\}\frac{C^2}{n}\right] \qquad \text{……(2.13)}$$

CONCLUSION

The results of the different estimators for the inverse of population mean may be easily seen to be the special cases of this study and so we can estimate the coefficient of variation efficiently.

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